

# Torsion Constraints in the Randall–Sundrum Scenario

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## Abstract

Torsion appears due to fermions coupled to gravity and leads to the strongest particle physics bounds on flat extra dimensions. In this work, we consider torsion constraints in the case of a warped extra dimension with brane and bulk fermions. From current data we obtain a  $3\sigma$  bound on the TeV–brane mass scale  $\Lambda_\pi \geq 2.2$  (10) TeV for the AdS curvature  $k = 1$  (0.01) in (reduced) Planck units. If Dirac or light sterile neutrinos reside on the brane, the bound increases to 17 (78) TeV.

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## 1 Introduction.

String theory requires the presence of extra dimensions. If some of these flat dimensions are large (submillimeter), the Standard Model (SM) hierarchy problem appears to be much milder [1]. The extra dimensions can also be warped [2], [3], i.e. generate a non–zero higher dimensional curvature. In this case, the hierarchy problem can be solved given an appropriate brane configuration in the higher dimensional space [2].

Current experiments require flat extra dimensions to be of order micrometer or smaller (see [4] for a recent review). These constraints come mainly from the phenomenology of the Kaluza–Klein (KK) excitations of the graviton. Light KK gravitons are emitted copiously in particle collisions and decays, and appear as missing energy and momentum [5],[6]. Also, exchange of virtual KK gravitons generates corrections to the SM predictions for collider observables and leads to additional constraints [6], [7]. The fundamental higher–dimensional Planck scale is bounded by these experiments to be greater than about 1 TeV. Astrophysical considerations increase this scale to about 50–70 TeV for two extra dimensions [8]. In the case of warped extra dimensions (the Randall–Sundrum model), the KK graviton modes are heavy but couple strongly to matter, so they can be detected as massive spin 2 resonances. Phenomenology of the Randall–Sundrum model has been studied extensively [9],[10] with the result that current experiments constrain the mass scale on the visible brane to be about 1 TeV or larger.

In these considerations, the gravitational connection  $\Gamma_{\mu\nu}^\alpha$  was assumed to be symmetric. Although this assumption leads to no inconsistencies, it is still an *assumption*. An alternative approach is to make no *a priori* assumption and find the gravitational connection through its equations of motion. This is known as the first order or Palatini formalism. In the absence of fermions, these two approaches are equivalent and lead to a symmetric connection. However, if fermions are present, an antisymmetric piece or torsion is induced in the first order formalism [11]. Torsion (at least classically) is not a dynamical degree of freedom and can be eliminated from the action via its equations of motion (for a recent review, see [12]). The result is a four-fermion interaction suppressed in the four-dimensional case by the Planck scale squared. The presence of torsion or, equivalently, this contact interaction is in agreement with the standard relativity tests [13] since its effect appears only in the presence of fermions and does not directly affect propagation of light. It is also worth mentioning that torsion is required in supergravity [14].

In four dimensions, the question whether torsion is present or not is only of academical interest because its effects are enormously suppressed. If extra space-time dimensions are present, the situation changes dramatically [15]. The torsion-induced contact interaction is suppressed only by the square of the fundamental scale which could be of order TeV. For flat extra dimensions, this enhancement has entirely different nature from that of the graviton mediated interactions. It results from the large fermionic spin density on the brane. This allows to obtain the strongest particle physics bounds on the higher dimensional fundamental scale. A global fit to the LEP/SLD electroweak observables yields a  $3\sigma$  bound [15]

$$M_S \geq 28 \text{ TeV} \quad (1)$$

for  $n = 2$ . It is worth emphasizing that this bound is obtained under a minimal set of assumptions. In particular, it is based on the standard gravity action and equations of motion for the connection. A more exotic possibility would be to assume propagating (dynamical) torsion [16], [17].

In the present work, we extend this analysis to the case of a warped extra dimension. We find that torsion effects provide a strong bound on the fundamental scale, yet not as severe as for flat extra dimensions. We also generalize these results to the case of bulk fermions.

## 2 Torsion in 4D (super)gravity.

In this section, we will introduce our notation and provide basic facts about torsion. We will follow the conventions of Ref.[18]. The metric is  $\eta_{\mu\nu} = (+ + + +)$  and the gamma matrices are hermitian. The Lorentz-covariant derivative of the Majorana spinor  $\chi$  is defined by

$$D_\mu \chi = \left( \partial_\mu + \frac{1}{2} \sigma^{ab} \omega_{\mu ab} \right) \chi . \quad (2)$$

with  $\sigma^{ab} = \frac{1}{4}(\gamma^a \gamma^b - \gamma^b \gamma^a)$ . The Lorentz connection  $\omega_{\mu ab}$  is antisymmetric in the last two indices and can always be written as a vierbein-dependent piece  $\omega_{\mu ab}^0(e)$  and the

contorsion  $\kappa_{\mu ab}$ :

$$\omega_{\mu ab} = \omega_{\mu ab}^0(e) + \kappa_{\mu ab} . \quad (3)$$

$\omega_{\mu ab}^0(e)$  can be obtained from the Einstein action in the absence of fermions via the equations of motion for  $\omega_{\mu ab}$  or, equivalently, by imposing the “tetrad postulate” that the fully covariant derivative (with the Christoffel connection) of the vierbein vanish,

$$\mathcal{D}_\mu^0 e_\nu^m \equiv \partial_\mu e_\nu^m + \omega_\mu^{0\ mn}(e)e_{n\nu} - \Gamma_{\nu\mu}^\alpha(g)e_\alpha^m = 0 . \quad (4)$$

The contorsion tensor accounts for matter effects. If the connection is considered an independent field and is found through its equations of motion, the contorsion tensor does not vanish in the presence of fermions. This is known as the Palatini or first-order formalism. It is advantageous in that it requires no *a priori* assumptions about the properties of the connection.

On the other hand, one may *assume* that the connection is always symmetric. This possibility is self-consistent and is motivated by the equivalence principle in its *very strong* form, i.e. that all gravity effects (up to higher order corrections) can be eliminated locally by an appropriate coordinate transformation [13]. Since contorsion (or torsion) is a tensor, it cannot be eliminated in this way. However, the assumption of local removability of “all” gravitational effects may be too strong. Indeed, the presence of torsion does not directly affect propagation of test particles and is in perfect agreement with the standard general relativity tests. To detect the presence of torsion would require a detailed investigation of the spin-gravitational effects with fermionic test particles. Thus, there is no compelling reason to assume that the connection stays symmetric in the presence of fermions. Also, we note that it is the first order formalism that leads to the standard description of supergravity, so torsion is present in locally supersymmetric theories [14].

Let us now consider how torsion arises in four dimensional gravity and supergravity.

**1. 4D Gravity.** The Lagrangian of gravity coupled to a Majorana fermion  $\chi$  is given by

$$\mathcal{L} = -\frac{1}{2\kappa^2}eR - \frac{1}{2}e\bar{\chi}\not{D}\chi , \quad (5)$$

where  $\kappa = \sqrt{8\pi}/M_{\text{Pl}}$ ,  $e = \det e_\mu^m$ , and  $R = e^{n\mu}e^{m\nu}R_{\mu\nu mn}$  with the curvature tensor

$$R_{\mu\nu}^{mn} = \partial_\mu \omega_\nu^{mn} + \omega_\mu^{mc}\omega_{\nu c}^n - (\mu \leftrightarrow \nu) . \quad (6)$$

Using the decomposition (3), we have

$$R = R(e) + e_n^\mu e_m^\nu D_{[\mu}^0 \kappa_{\nu]}^{mn} - \kappa_{\mu\rho\nu}\kappa^{\nu\rho\mu} + (\kappa_\nu^{\nu\rho})^2 , \quad (7)$$

where  $[\mu\nu] \equiv \mu\nu - \nu\mu$  and  $R(e)$  is built on  $\omega_{\mu ab}^0(e)$ . In the second term of this equation, the Lorentz-covariant derivative  $D_\mu^0$  built on  $\omega_{\mu ab}^0(e)$  can be replaced by the fully covariant derivative  $\mathcal{D}_\mu^0$  built on  $\omega_{\mu ab}^0(e)$  and the Christoffel symbols  $\Gamma_{\nu\mu}^\alpha(g)$ . Using  $\mathcal{D}_\mu^0 e_\nu^m = 0$ , it is easy to see that this term is a total divergence. Therefore, the Einstein action has an algebraic dependence on the contorsion tensor. It is easy to see that the contorsion derivatives do not appear anywhere and thus contorsion is a non-propagating field which can be eliminated algebraically.

The Einstein action has a quadratic dependence on the contorsion while the fermion action depends on it linearly. Thus, varying the action with respect to  $\kappa^{\nu\rho\mu}$  gives\*

$$\kappa_{\mu\nu\rho} - \kappa_{\rho\nu\mu} = -\frac{e\kappa^2}{4}\epsilon_{\mu\nu\rho\sigma}\bar{\chi}\gamma^5\gamma^\sigma\chi. \quad (8)$$

This equation can be solved for contorsion:

$$\kappa_{\mu\nu\rho} = -\frac{e\kappa^2}{8}\epsilon_{\mu\nu\rho\sigma}\bar{\chi}\gamma^5\gamma^\sigma\chi. \quad (9)$$

The torsion tensor is defined as the antisymmetric part of the connection  $\Gamma_{\mu\nu}^\alpha$  which is related to the spin-connection via  $\partial_\mu e_\nu^m + \omega_\mu^{mn}e_{n\nu} - \Gamma_{\nu\mu}^\alpha e_\alpha^m = 0$ ,

$$S_{\mu\nu}^\alpha = \frac{1}{2}(\Gamma_{\mu\nu}^\alpha - \Gamma_{\nu\mu}^\alpha) = \frac{1}{2}(-\kappa_{\mu}{}^\alpha{}_\nu + \kappa_{\nu}{}^\alpha{}_\mu). \quad (10)$$

Clearly, the torsion tensor is proportional to  $D_\mu e_\nu^m - D_\nu e_\mu^m$ . From Eq.9 we obtain

$$S_{\mu\nu\alpha} = -\frac{e\kappa^2}{8}\epsilon_{\mu\nu\alpha\sigma}\bar{\chi}\gamma^5\gamma^\sigma\chi. \quad (11)$$

We see that (con)torsion vanishes outside matter distribution and is completely antisymmetric if only spin 1/2 fermions are present.

Using Eqs.7 and 9, the contorsion tensor can be eliminated from the action. This results in the four-fermion axial interaction

$$\Delta\mathcal{L} = \frac{3e\kappa^2}{64}(\bar{\chi}\gamma^5\gamma^\sigma\chi)^2. \quad (12)$$

Interaction of this form is specific to torsion and cannot be induced by the graviton exchange. It is repulsive for aligned spins. Indeed, in the non-relativistic limit the corresponding Hamiltonian reads

$$\Delta\mathcal{H} = -\frac{3e\kappa^2}{64}(\bar{\chi}\gamma^5\gamma^\sigma\chi)^2 \longrightarrow \frac{3e\kappa^2}{64}(u^\dagger \vec{\sigma} u)^2, \quad (13)$$

where  $u$  is a two-component non-relativistic spinor.

In the case of many Majorana fields, Eq.12 generalizes to

$$\Delta\mathcal{L} = \frac{3e\kappa^2}{64}\left(\sum_i \bar{\chi}_i\gamma^5\gamma^\sigma\chi_i\right)^2. \quad (14)$$

It is useful to convert this interaction into that for the Dirac fermions. Expressing  $\Psi = P_L\chi_1 + P_R\chi_2$  with  $P_{L,R}$  being the left and right projectors and using  $\bar{\chi}\gamma^\mu\chi = 0$ , we have<sup>†</sup>

$$\Delta\mathcal{L} = \frac{3e\kappa^2}{16}\left(\sum_i \bar{\Psi}_i\gamma^5\gamma^\sigma\Psi_i\right)^2, \quad (15)$$

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\*All  $\epsilon$ -tensors are assumed to take on the values  $\pm 1, 0$ . Thus, lowering the world indices of the  $\epsilon$ -tensor is accompanied by dividing it by  $\det g_{\mu\nu}$  and  $\epsilon^{\mu\nu\rho\sigma} = (\det e_m^\mu)^{-1}e_m^\mu e_n^\nu e_k^\rho e_l^\sigma \epsilon^{m n k l}$ .

<sup>†</sup>In the notation of Ref.[], the coefficient of this contact interaction would be  $3e\kappa^2/32$  since  $\kappa^2$  defined here is twice as small.

where the sum now runs over Dirac fermions. The most peculiar feature of this interaction is that it is completely universal for all of the spin 1/2 fermions. In the case of the Standard Model, it possesses the maximal possible symmetry  $U(45)$  acting on the 45 Weyl spinors [19]. This universality stems from the fact that torsion couples to the spin density. In contrast, the four-fermion interactions induced by the graviton exchange would depend on the energy and masses of the fermions since the graviton couples to the energy-momentum tensor. The interaction (15) is a truly gravitational effect which can hardly be “counterfeited” by other physics.

Whereas in the case of gravity the presence of torsion is logical yet optional, supergravity theory requires torsion [14]. Let us now consider how the above equations get modified in supergravity.

**2. 4D Supergravity.** The Lagrangian of supergravity coupled to a chiral supermultiplet in the first order formulation is given by [20]

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2\kappa^2}eR - \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\Psi}_\mu\gamma_5\gamma_\nu D_\rho\Psi_\sigma - \frac{1}{2}e\bar{\chi}\not{D}\chi - \frac{1}{16}e\kappa^2(\bar{\chi}\gamma^5\gamma^\sigma\chi)^2 \\ & + \text{connection independent terms} . \end{aligned} \quad (16)$$

Here  $\Psi_\nu$  is the gravitino field and

$$D_\mu\Psi_\nu = \left(\partial_\mu + \frac{1}{2}\sigma^{ab}\omega_{\mu ab}\right)\Psi_\nu . \quad (17)$$

Even though this derivative is not covariant in the world indices, its *curl* is, so the above action is indeed invariant under coordinate transformations. The supergravity multiplet, the spin-connection, and the matter multiplet are considered to be independent. Each of them has its own supersymmetry transformation (see e.g.[20]). An interesting feature of the above Lagrangian is that supersymmetry requires the presence of the four-fermion axial interaction which appears independently from the one induced by torsion.

Variation of the action with respect to contorsion is done the same way discussed above except now there is an additional contribution from the gravitino field. The field equation for contorsion is

$$\kappa_{\mu\nu\rho} - \kappa_{\rho\nu\mu} = \frac{\kappa^2}{2}\bar{\Psi}_\mu\gamma_\nu\Psi_\rho - \frac{e\kappa^2}{4}\epsilon_{\mu\nu\rho\sigma}\bar{\chi}\gamma^5\gamma^\sigma\chi . \quad (18)$$

Solving for contorsion, we get

$$\kappa_{\mu\nu\rho} = \frac{\kappa^2}{4}\left(\bar{\Psi}_\mu\gamma_\nu\Psi_\rho + \bar{\Psi}_\nu\gamma_\mu\Psi_\rho - \bar{\Psi}_\mu\gamma_\rho\Psi_\nu\right) - \frac{e\kappa^2}{8}\epsilon_{\mu\nu\rho\sigma}\bar{\chi}\gamma^5\gamma^\sigma\chi . \quad (19)$$

Note that in the presence of spin 3/2 particles the contorsion and torsion tensors are no longer completely antisymmetric. Eliminating contorsion from the action, we get the same four-fermion interaction as in the case of gravity plus additional terms involving the gravitino. In addition to this torsion-induced interaction, we have the original four-fermion term in Eq.16. The final result is

$$\Delta\mathcal{L} = -\frac{e\kappa^2}{64}(\bar{\chi}\gamma^5\gamma^\sigma\chi)^2 + \text{gravitino terms} . \quad (20)$$

It is remarkable that, compared to the previous case, not only the numerical coefficient has changed but also the interaction has flipped its sign. That means that if we start with supergravity, break it spontaneously, and integrate out the superpartners, the residual four-fermion interaction will be different from that of gravity. Another interesting feature of supergravity is that if we start with a vector supermultiplet coupled to supergravity, the resulting four-gaugino interaction turns out to be  $\Delta\mathcal{L} = 3e\kappa^2/16 (\bar{\lambda}\lambda)^2$  instead of (20).

The torsion-induced interaction in four dimensions is suppressed by  $M_{\text{Pl}}^2$  and thus is undetectable. However, if there are additional large space-time dimensions, the situation changes drastically and the torsion effects become not only visible but even dominant [15]. In the next section, we consider torsion effects in the case of a warped extra dimension, i.e. the Randall-Sundrum model.

### 3 Torsion in the Randall-Sundrum model.

The Randall-Sundrum setup provides an attractive way to generate a hierarchy between the electroweak and the Planck scales [2]. The basic idea is to start with a five-dimensional anti-de Sitter (AdS) space-time with two 3-branes. With a special choice of the vacuum energies in the bulk and on the branes, this configuration can be made stable. The distance between the branes is determined by a VEV of the radion field [21]. Then, the geometrical warp factor representing the overlap of the graviton wave function with the observable brane is responsible for the apparent weakness of gravity.

The AdS metric

$$ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \quad (21)$$

is induced by the gravitational fields of the branes. Here  $y$  ( $-\pi r < y \leq \pi r$ ) parametrizes the orbifold extra dimension of radius  $r$  and  $y \equiv -y$ . This metric is a solution to the Einstein equations if<sup>‡</sup>

$$\begin{aligned} S_{\text{bulk}} &= - \int d^5x \, \hat{e} \left( \frac{1}{2} M_5^3 R + \Lambda \right) , \\ S_{\text{brane}} &= - \int d^5x \, e \left[ \delta(y) (\Lambda_1 + \mathcal{L}_1) + \delta(y - \pi r) (\Lambda_2 + \mathcal{L}_2) \right] , \end{aligned} \quad (22)$$

with

$$\begin{aligned} \Lambda &= -6M_5^3 k^2 , \\ \Lambda_1 &= -\Lambda_2 = -\frac{\Lambda}{k} . \end{aligned} \quad (23)$$

Here  $1/k$  is the AdS curvature radius and  $M_5$  is the five-dimensional Planck mass. They are related to the four-dimensional reduced Planck mass  $\tilde{M}_{\text{Pl}} = M_{\text{Pl}}/\sqrt{8\pi}$  via

$$\tilde{M}_{\text{Pl}}^2 = \frac{M_5^3}{k} (1 - e^{-2\pi k r}) . \quad (24)$$

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<sup>‡</sup>We use the notation of Ref.[22].

We note that the gravitational constant  $\kappa$  introduced in the previous section is given by  $\kappa = 1/\tilde{M}_{\text{Pl}}$ . With the above metric it can be easily shown that the natural mass scale at  $y = 0$  is  $M_{\text{Pl}}$  while that at  $y = \pi r$  is  $M_{\text{Pl}} e^{-\pi k r}$ . Thus, with  $kr \simeq 12$ , we obtain a TeV scale at  $y = \pi r$  (the visible brane). The mass hierarchy in this case has a geometrical origin – it appears owing to the AdS metric in the five-dimensional space.

Let us now consider how torsion arises in this setup. We will discuss separately torsion effects induced by brane fermions and fermions propagating in the bulk.

**i. Brane fermions.** The visible brane contains Majorana fermions (which can be converted into Dirac ones) with the kinetic terms

$$S_{\text{brane ferm.}} = \int d^5x \delta(y - \pi r) \left[ -\frac{1}{2} e \sum_i \bar{\chi}_i \not{D} \chi_i \right]. \quad (25)$$

These are the Standard Model fermions localized on the brane. To find the equations of motion for the connection, we vary the action with respect to contorsion noting that Eq.7 is valid for an arbitrary number of space-time dimensions. The result is

$$\kappa_{\mu\nu\rho} = -\frac{e \delta(y - \pi r)}{8M_5^3} \epsilon_{\mu\nu\rho\sigma} \sum_i \bar{\chi}_i \gamma^5 \gamma^\sigma \chi_i. \quad (26)$$

Here we have used  $\hat{e} = e$  in the AdS background. The other torsion components are zero. Eliminating non-propagating contorsion from the action, we obtain the following axial contact interaction on the visible brane:

$$\Delta\mathcal{L} = \frac{3e \delta(0)}{64M_5^3} \left( \sum_i \bar{\chi}_i \gamma^5 \gamma^\sigma \chi_i \right)^2. \quad (27)$$

The arising delta-function singularity is due to the implicit assumption that the brane is infinitely thin. (Con)torsion is proportional to the fermionic spin-density, which from five-dimensional point of view becomes infinite on the brane. In practice, the delta-function should be regularized to account for a finite brane width:

$$\delta(0) \rightarrow \frac{1}{2\pi} \int_{-M_5}^{M_5} dk = \frac{M_5}{\pi}, \quad (28)$$

where we have taken the five-dimensional Planck mass as a natural cut-off so that the brane width is of order  $M_5^{-1}$ .

The metric on the visible brane is given by  $e^{-2k\pi r} \eta_{\mu\nu}$ . Thus, the fermion kinetic terms in Eq.25 are not canonically normalized:

$$-\frac{1}{2} e \bar{\chi} \not{D} \chi = -\frac{1}{2} e^{-2k\pi r} \eta^{\mu\nu} \bar{\chi} \gamma_\mu D_\nu \chi. \quad (29)$$

Rescaling  $\chi \rightarrow \chi e^{k\pi r}$  and using Eq.28, we obtain the following four-fermion interaction for the properly normalized fields:

$$\Delta\mathcal{L} = \frac{3}{64\pi (e^{-k\pi r} M_5)^2} \left( \sum_i \bar{\chi}_i \gamma^5 \gamma^\sigma \chi_i \right)^2 = \frac{3}{16\pi (e^{-k\pi r} M_5)^2} \left( \sum_i \bar{\Psi}_i \gamma^5 \gamma^\sigma \Psi_i \right)^2, \quad (30)$$

where  $\Psi_i$  are the Dirac brane fermions. As we see, the torsion-induced interaction is suppressed only by the TeV scale, unlike in the 4D case. The reason is that  $e^{-k\pi r}M_5$  can be viewed as the fundamental scale [2]. Also, the large five-dimensional spin-density on the brane plays an important role. If we allow some of the fermions to propagate in the bulk, the torsion-induced interaction of their zero modes will be suppressed by the “volume” factor  $kr \sim \mathcal{O}(10)$ . This differs from the case of flat extra dimensions in which case large spin-density on the brane was the only reason for enhancement of the torsion-induced interaction.

We note that the Kaluza-Klein graviton exchange also generates four-fermion interactions. These are however further suppressed by  $E^2/(e^{-k\pi r}M_5)^2$  with  $E$  being the typical energy of the process since the graviton couples to the energy-momentum tensor [6]. Furthermore, the graviton exchange can induce neither axial nor universal contact interaction.

The axial interaction (30) induces significant vertex corrections to the Z couplings to fermions and is strongly constrained by the electroweak precision data. The crucial point is that it is *universal* for all of the fermions and contains no variable parameters apart from the mass scale. The global fit to the LEP/SLD observables was performed in Ref.[15]. It was found that this interaction is excluded at the  $2\sigma$  level by classical statistical analysis and is allowed only at the  $3\sigma$  level. This occurs mainly because (30) increases the  $Z \rightarrow \nu\bar{\nu}$  width whereas its experimental value is about  $2\sigma$  smaller than the Standard Model prediction<sup>§</sup>. The major effect comes from the vertex correction with the top quark in the loop. The corresponding shift in the Z-couplings is then  $\delta h_L = -\delta h_R = 3N_c m_t^2/(64\pi^3 M_*^2) \ln M_*/m_t^2$ , where for brevity we denoted  $M_* \equiv e^{-k\pi r}M_5$ . The resulting  $3\sigma$  bound is  $e^{-k\pi r}M_5 \geq 2.2$  TeV, or in the notation of Ref.[9],

$$\Lambda_\pi \left( \frac{k}{\tilde{M}_{\text{Pl}}} \right)^{1/3} \geq 2.2 \text{ TeV} , \quad (31)$$

where  $\Lambda_\pi = \tilde{M}_{\text{Pl}} e^{-k\pi r}$  and  $\tilde{M}_{\text{Pl}}^2 \simeq M_5^3/k$ . The Randall-Sundrum solution is valid for  $k \leq \tilde{M}_{\text{Pl}}$ . If  $k \simeq \tilde{M}_{\text{Pl}}$ , we have  $\Lambda_\pi \geq 2.2$  TeV, while for  $k \simeq 0.01\tilde{M}_{\text{Pl}}$  the constraint becomes significantly stronger:  $\Lambda_\pi \geq 10$  TeV.

An additional (tree-level) constraint on the interaction (30) can be obtained from the OPAL measurements of the differential cross sections  $e^+e^- \rightarrow f\bar{f}$  [24]. These imply

$$\Lambda_\pi \left( \frac{k}{\tilde{M}_{\text{Pl}}} \right)^{1/3} \geq 0.8 \text{ TeV} \quad (32)$$

at the 95% confidence level. Constraints from DIS data, Drell-Yan production, etc. are weaker [15].

The  $3\sigma$  bound (31) is quite significant. For instance, with  $k \sim \tilde{M}_{\text{Pl}}$ , it is stronger than the 95% C.L. bound obtained from collider data in Ref.[9] by about a factor of five. Yet, torsion constraints on the fundamental scale are even more severe in the case of flat extra dimensions, e.g.  $M_S \geq 28$  TeV for  $n = 2$ . The reason is that for flat extra

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<sup>§</sup>On possible relevance of supersymmetry to this deviation see Ref.[23].



dimensions the spin–density on the brane is more singular because of the large hierarchy between the compactification scale and the fundamental scale (apart from the fact that here we constrain the “reduced” fundamental scale). In the Randall–Sundrum scenario, this hierarchy is only one-two orders of magnitude, so the effect of  $\delta(0)$  is not as significant.

Finally, there is a strong astrophysical bound on the interaction (30). If the Dirac or light ( $m_{\nu_s} \ll 50$  MeV) sterile neutrinos live on the brane, torsion induces

$$\Delta\mathcal{L} = \frac{6}{16\pi(e^{-k\pi r}M_5)^2} \bar{q}\gamma^5\gamma^\sigma q \bar{\nu}_R\gamma^5\gamma_\sigma\nu_R. \quad (33)$$

A quark–neutrino contact interaction of this type provides a new channel of energy drain during neutron star collapse and is severely constrained by the supernova data [25]. The corresponding bound from SN 1987A is

$$\Lambda_\pi \left( \frac{k}{\tilde{M}_{\text{Pl}}} \right)^{1/3} \geq 17 \text{ TeV}. \quad (34)$$

As we will see below, this bound relaxes if we allow neutrinos to propagate in the bulk.

We note that the only source of uncertainty in the above (classical) calculations is the brane width or, equivalently, regularization of the delta function. Our bounds are inversely proportional to the square root of the brane width and, thus, are not very sensitive to this source of uncertainty. If the brane width exceeds  $M_5^{-1}$  considerably, the common  $\delta$ -function description of the brane breaks down as the brane width would be comparable to the compactification radius. In addition, there are, of course, ever-present quantum corrections. For instance, the fermion–torsion coupling  $S_\mu \bar{\Psi}\gamma^5\gamma^\mu\Psi$ , where  $S_\mu \propto \epsilon_{\mu\nu\rho\sigma}\kappa^{\nu\rho\sigma}$ , induces the torsion kinetic terms  $\partial_{[\mu}S_{\nu]}$  via the fermion loop. Thus, torsion becomes dynamical and can propagate along the brane. Our approximation corresponds to shrinking the torsion propagator into a point. Since the torsion mass coming from the scalar curvature is of order the cut-off scale, this approximation is very good at low energies.

**ii. Bulk fermions.** Let us now consider torsion effects due to bulk fermions. A straightforward modification of the kinetic terms (25)  $\gamma^\mu D_\mu \rightarrow \gamma^\mu D_\mu + \gamma^5 D_5$  leads to the following equations of motion for contorsion:

$$\begin{aligned} \kappa_{\mu\nu\rho} &= -\frac{e}{4M_5^3}\epsilon_{\mu\nu\rho\sigma}\sum_i\bar{\Psi}_i\gamma^5\gamma^\sigma\Psi_i, \\ \kappa_{5\nu\rho} &= -\frac{1}{2M_5^3}\sum_i\bar{\Psi}_i\gamma_5\sigma_{\nu\rho}\Psi_i, \\ \kappa_{55\rho} &= 0, \end{aligned} \quad (35)$$

where  $\Psi_i$  are the five-dimensional Dirac fermions. Note that  $\kappa_{5\nu\rho}$  is completely antisymmetric since  $\gamma_5\sigma_{\nu\rho} = \gamma_{[\nu}\gamma_\rho]/12$ , so the other components of the contorsion tensor with two four-dimensional indices are given by permutations. It is interesting to note that new (tensor) structures have appeared compared to the 4D fermion case.

To obtain chiral fermions in four dimensions, the Dirac spinor is taken to obey the orbifold boundary condition  $\Psi(-y) = \pm\gamma_5\Psi(y)$  (see e.g. [26]). Since  $\bar{\Psi}\gamma_5\sigma_{\nu\rho}\Psi$  is odd

under  $y \rightarrow -y$ ,  $\kappa_{5\nu\rho}$  vanishes at the orbifold fixed points. Thus, the torsion-induced interaction on the visible brane has the same structure as before, i.e. an axial vector squared.

The observed fermions correspond to the massless (zero) modes of the 5D fermions. The properly normalized zero modes are given by [22]

$$\Psi(x, y) = \frac{e^{(1/2-c)k|y|}}{\sqrt{2\pi r}N_0}\Psi(x) , \quad (36)$$

with  $N_0$  being the normalization factor,

$$N_0^2 = \frac{e^{2\pi kr(1/2-c)} - 1}{2\pi kr(1/2 - c)} . \quad (37)$$

Here the constant  $c$  indicates the localization of the zero mode in the AdS space: for  $c < 1/2$  the zero mode peaks on the TeV-brane, for  $c > 1/2$  it is localized towards the Planck ( $y = 0$ ) brane, and for  $c = 1/2$  it is constant in the  $y$ -direction.

For clarity, we assume  $c = 1/2$  in which case  $N_0 = 1$  and  $\Psi(x, y) = \frac{1}{\sqrt{2\pi r}}\Psi(x)$ . Then, the torsion-induced interaction is given by

$$\Delta\mathcal{L} = \frac{3}{32\pi(e^{-k\pi r}M_5)^2rM_5}\left(\sum_i\bar{\Psi}_i\gamma^5\gamma^\sigma\Psi_i\right)^2 , \quad (38)$$

Compared to the previous case, the delta function  $\delta(0) = M_5/\pi$  got replaced by the volume factor  $(2\pi r)^{-1}$ . This results in an order of magnitude suppression. Since  $M_5r \simeq 12M_5/k = 12(\tilde{M}_{\text{Pl}}/k)^{2/3}$  and  $(e^{-k\pi r}M_5)^2 = \Lambda_\pi^2(\tilde{M}_{\text{Pl}}/k)^{-2/3}$ , we have

$$\Delta\mathcal{L} \simeq \frac{1}{128\pi\Lambda_\pi^2}\left(\sum_i\bar{\Psi}_i\gamma^5\gamma^\sigma\Psi_i\right)^2 . \quad (39)$$

The resulting bounds on  $\Lambda_\pi$  are about a factor of 5 weaker than in the case of localized fermions (with  $k \simeq \tilde{M}_{\text{Pl}}$ ). As mentioned above, this is to be contrasted with the case of flat extra dimensions: if the SM fermions are allowed to propagate in the extra dimensions, no appreciable torsion constraint can be obtained. We note that the bounds relax further if fermions are localized closer to the  $y = 0$  brane ( $c > 1/2$ ).

Finally, the interaction of two brane and two bulk fermions is the same as if they all were bulk fermions of different species and is given by Eq.39. This possibility is interesting since a coupling between a neutral bulk fermion and the brane neutrino and Higgs induces a naturally small Dirac neutrino mass for flat extra dimensions [27]<sup>¶</sup>. In this case torsion effects require  $M_S \geq 28$  TeV (ADD,  $n = 2$ ) and  $\Lambda_\pi \geq 3$  TeV (RS).

## 4 Discussion and conclusions.

We have considered the effects of torsion in the Randall-Sundrum model with two branes. Our analysis was based on the first order formalism. That is, instead of making an *a priori*

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<sup>¶</sup>One should keep in mind that this scenario is strongly constrained by the supernova data [28].

assumption that the gravitational connection is symmetric, we determined the properties of the connection via its equations of motion. This resulted in a universal axial contact interaction suppressed by a TeV scale. Assuming that the SM fermions are confined to the brane, current LEP/SLD electroweak data constrain the mass scale of the visible brane  $\Lambda_\pi$ :

$$\Lambda_\pi \left( \frac{k}{\tilde{M}_{\text{Pl}}} \right)^{1/3} \geq 2.2 \text{ TeV} . \quad (40)$$

If Dirac or light sterile neutrinos also reside on the brane, this bound increases to 17 TeV from the supernova SN1987 observations. The bounds relax by roughly a factor of 5 if the fermions are allowed to propagate in the bulk.

It would be interesting to extend this analysis to the supersymmetric Randall–Sundrum model. Important steps for the case of pure supergravity have been made in this direction in Ref.[29] (see also [22]), however the four–fermion terms were neglected. It is curious that if the sign of the contact interaction flips upon supersymmetrizing, as it does in four dimensions (Eq.20), this interaction will be preferred by the electroweak data and will help rectify the deviation in the invisible Z width. The other constraints will however persist.

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